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Cylindrically Symmetric Model in Presence of Electromagnetic Field in General Relativity

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Abstract :

In this paper we have investigated cylindrically model in presence of electromagnetic field. Taking a suitable metric we have also found and discussed several physical and geometric properties e.g., pressure, density, scalar of expansion and shear tensor. It is found that the electric field, the four current, the fluid density, pressure and scalar of expansion all start with the finite values at the initial singularity (t = 0) and tend to zero.

Key Word : Cylindrical symmetry, electromagnetic field pressure, density, expansion, shear.

1. INTRODUCTION

Keen interest has been shown by many authors towards the study of cylindrically symmetric cosmological models in general relativity in recent years [4(a), 7, 19, 11(a), (b), 12(a), 20]. A cosmological model in presence of magnetic field has been studied by Zeldovich [21] and later by Thorne [15]. Shikin [13] also constructed a uniform axially symmetric solution (model) of Einstein's gravitational equation and Maxwell's equations in the case of propagation by a completely ideal substance in the presence of magnetic field directed along the axis of symmetry. Magnetic field in stellar bodies was also discussed by Monoghan [10]. Gravitational collapse of a magnetic star was studied by Ginzburg [2]. Seymour [12] also derived some models of the galactic magnetic field. Jacoba [5, 6] has studied the behaviour of the general Bianchi – type I cosmological model in the presence of a spatially homogeneous magnetic field. This problem has been studied again by De [1] with a different approach. This work has been further extended by Tupper [17] to include Einstein – Maxwell fields in which the electric field is non-zero. He has also interpreted certain type VI cosmologies with electromagnetic field (Tupper [18]). Roy and Prakash [11], taking the cylindrically symmetric metric of Marder [9], have constructed a spatially homogeneous cosmologicval model in the presence of an incident magnetic field which is also anisotropic and non-degenerate petrov type-I. Singh and Yadav [14] constructed a spatially homogeneous cosmological model assuming the energy momentum tensor to be that of a perfect fluid with an electromagnetic field.

The present paper provides a cylindrically symmetric cosmological model with electromagnetic field which has a four current which is either zero or space like. Taking a suitable metric we have also calculated various physical and geometrical properties e.g. pressure, density, scalar of expansion and components of shear tensor. We have also discussed behaviour of a test particle in the model. It is found that the electric field, the four current, the field density, the pressure and scalar of expansion all start with infinites values at the initial singularity (t = 0) and tends to zero when t tends to infinity.

2. THE FIELD EQUAITONS AND THEIR SOLUTIONS :

We consider the most general cylindrically symmetric space-time in the form given by

(4.2.1) $ds^2 = A^2(dt^2 - dx^2) - B^2 dy^2 - C^2 dz^2$

Where the metric potentials A, B, C are functions of time t alone. The distribution consists of a perfect fluid and an electromagnetic field. The energy momentum tensor of the composite field is assumed to be the sum of the corresponding energy momentum tensors. Thus

(2.2) $R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R + \wedge g_{\mu\upsilon} = -k\left[\left(\rho + P)\right) u_\mu u_\upsilon - P g_{\mu\upsilon} + E_{\mu\upsilon}\right]$

(2.3) $E_{\mu\upsilon} = -g^{k\ell} F_{\mu k} F_{\mu\upsilon\ell} + \frac{1}{4} g_{\mu\upsilon} F_{\alpha\beta} F^{\alpha\beta}$

(2.4) $E_{[\mu\upsilon:\sigma]} = 0$

(2.5) $F^{\mu\upsilon}_{;\upsilon} = J^\mu$

where p and ρ pressure and density respectively of the distribution, $E_{\mu\upsilon}$ is the electromagnetic energy momentum tensor, $F_{\mu\upsilon}$ is the electromagnetic field tensor, J^μ is the current 4-vector, $\wedge$ is the cosmological constant and u_μ is the flow vector satisfying

(2.6) $g_{\mu\upsilon} u_\mu u_\upsilon = 1$

The co-ordinates are chosen to be comoving so that

(2.7) $u^\mu = \left(0, 0, 0 \frac{1}{A}\right)$

and we label the co-ordinates

$$(X, Y, Z, t) \equiv \left(X^1, X^2, X^3, X^4\right)$$

we assume the electromagnetic field to be in the direction of X-axis so that F_{14} and F_{23} are the only non-vanishing components of the field tensor $F_{\mu\upsilon}$. We write

(2.8) $F_{14}^2 A^{-4} + F_{23}^2 B^{-2} C^{-2} = M^2$

The equation (2.2) may be written as

(2.9) $\frac{2}{A^2}\left[\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4 C_4}{AC} - \frac{A_4 B_4}{AB} - \frac{A_4^2}{A^2}\right] - 2\wedge$

$$= -K[M^2 + (\rho + 3P)],$$

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(2.10) $$-\frac{2}{A^2}\left[\frac{A_{44}}{A}+\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}-\frac{A_4^2}{A^2}\right]+2\wedge$$

$$=-K[-M^2+(\rho-P)],$$

(2.11) $$-\frac{2}{A^2}\left[\frac{B_{44}}{B}+\frac{B_4C_4}{BC}\right]+2\wedge=-K[M^2+(\rho-P)],$$

(2.12) $$-\frac{2}{A^2}\left[\frac{C_{44}}{C}+\frac{B_4C_4}{BC}\right]+2\wedge=-K[M^2+(\rho-P)]$$

where the suffix 4 after the symbols A, B, C denotes ordinary differentiation w.r.t. time t. These equations show that M^2, ρ, P are each functions of t only, and it then follows from equations (2.4) and (2.8) that F_{23} is a constant and F_{14} is a function of t only i.e.

(2.13) $$F_{23}=k, F_{14}=\pm A^2(M^2-K^2B^{-2}C^{-2})^{1/2}$$

where k is a constant.

The case when F_{14} = 0, which implies J^μ = 0, we get the model due to Roy and Prakash [11]. We here assume that $F_{14}\neq 0$, and find the only non-vanishing components of J^μ to be

(2.14) $$J^1=\pm\frac{1}{A^2BC}\frac{\partial}{t}\left[BC(M^2-k^2B^{-2}C^{-2})^{1/2}\right]$$

Equation (2.14_ shows that J^μ is spacelike, unless $M^2 = 1B^{-2}C^{-2}$ where *1* is a constant in which case J^μ = 0. The 4 – current J^μ is in general the sum of the convection current and conduction current (Licknerowioz [8]and Greenburg [3]).

(2.15) $$J^\mu=\sigma_0u^\mu+\lambda_{\mu\upsilon}F^{\mu\upsilon}$$

where σ_0 is the rest charge density and λ is the conductivity. In the case considered here we have $\sigma_0 = 0$ i.e., magnetohydrodynamics. From equations (2.13), (2.14) and (2.15) we find that the conductivity is given by

(2.16) $$\lambda=-\frac{1}{A}D_4D^{-1}$$

where $D=BC(M^2-k^2B^{-2}C^{-2})^{1/2}$

The requirement of positive conductivity in (2.16) puts further restrictions on A, B, C. Hence in the magnetohydro-dynamics case metric function are restricted not only by the field equations and energy conditions (Hawking and Penrose [4]) they are also restricted by the requirement that the conductivity be positive for a realistic model.

Finally we illustrate the situation described here by an example. Consider the space time with metric

(2.17) $ds^2 = t^{16\phi^2}(dx^2 - dt^2) + t^{4\phi}(dt^2 + dz^2)$

which is obtained from metric (2.1) when

$$A = t^{8\phi^2}, B = C = t^{2\phi}$$

where ϕ being an arbitrary constant parameter.

Equations (2.9) – (2.12) lead to

(2.18) $M^2 = 2\phi(8\phi + 1)(2\phi - 1)t^{-16\phi^2 - 2}$

(2.19) $\rho = \phi(6\phi - 16\phi^2 - 1)t^{-16\phi^2 - 2} + \wedge$

(2.20) $p = \phi(16\phi^2 + 2\phi - 3)t^{-16\phi^2 - 2} - \wedge$

Clearly M^2, ρ, p all are decreasing function of time.

From equation (2.18)

(2.21) $0 > \phi > -\frac{1}{8}$

when $\wedge = 0$ and $p > 0$

Again when

$$\phi = \pm\frac{1}{2\sqrt{2}}$$

Then it is the case of disordered radiation i.e.,

$$\rho = 3p$$

Also when

$$\phi = \frac{1}{16} \pm \frac{\sqrt{17}}{16}$$

Then it is the case of stiff matter i.e.,

$$\rho = p.$$

However these value of ϕ are not admissible to have a model of physical significance. It is seen that the model satisfies the dominant energy condition (Hawking and Phenrose [4]) and the fluid energy condiciton.

$$\rho + p > 0$$

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The electromagnetic field components are

(2.22) $$F_{14} = \pm t^{16\phi^2-2\phi}\left(t^{-16\phi^2+8\phi-2}\phi(8\phi+1)(2\phi-1)-k^2\right)^{1/2}$$

and the magnitude of the electromagnetic field is restricted by

(2.23) $$K^2 < 2\phi(2\phi-1)(8\phi+1)t^{-16\phi^2+8\phi-2}$$

The non zero component of the current four vector is

(2.24) $$J^1 = \pm\phi(2\phi-1)(8\phi+1)(16\phi^2-8\phi+2)t^{-24\phi^2+4\phi-3} \times\left[2\phi(2\phi-1)(8\phi+1)t^{-16\phi^2+8\phi-2}-k^2\right]^{-1/2}$$

also the conductivity for the model is given by

(2.25) $$\lambda = \phi(2\phi-1)(8\phi+1)(16\phi^2-8\phi+2)t^{-24\phi^2+8\phi-3} \times[2\phi(2\phi-1)(8\phi+1)t^{-16\phi^2+8\phi-2}-k]^{-1}$$

Scalar of expansion defined by

(2.26) $$\theta = u^{\mu};\mu$$

is given by

(2.27) $$\theta = 4\phi t^{-8\phi^2-1}[1-2\phi]$$

Hence the electric field, the 4-current, the fluid density, the pressure and scalar of expansion all start with infinite values at the initial singularity (t = 0) and tends to zero when $t \to \infty$.

The components of the shear tensor defined by

(2.28) $$\sigma_{\mu\upsilon} = \frac{1}{2}(4_{\mu;\upsilon} + u\upsilon;\mu) - \frac{1}{3}\theta(g_{\mu\upsilon} - u_{\mu}u_{\upsilon})$$

are

(2.29) $$\left|\begin{array}{l} 6_{11} = 4\phi(4\phi-1)t^{8\phi^2-1} \\ 6_{22} = 6_{33} = 2\phi(4\phi-3)t^{8\phi^2+4\phi-1} \\ 6_{44} = 4\phi(1-2\phi)t^{8\phi^2-1} \end{array}\right.$$

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3. BEHAVIOUR OF A TEST PARTICLE IN THE MODEL :

The equation of geodesic viz.

(3.1) $$\frac{d^2x^i}{ds^2} + \Gamma^i_{jk} \frac{dx^k}{ds}\frac{dx^j}{ds} = 0$$

(for the metric (2.17) when I = 1, 2, 3, 4 are given by

(3.2) $$\frac{d^2x}{ds^2} + 4\phi^2 t^{-1} \frac{dx}{ds}\frac{dt}{ds} = 0$$

(3.3) $$\frac{d^2y}{ds^2} + 2\phi t^{-1} \frac{dy}{ds}\frac{dt}{ds} = 0$$

(3.4) $$\frac{d^2z}{ds^2} + 2\phi t^{-1} \frac{dz}{ds}\frac{dt}{ds} = 0$$

(3.5) $$\frac{d^2t}{ds^2} + 8\phi^2 t^{-1}\left(\frac{dx}{ds}\right)^2 - 2\phi t^{-16\phi^2+4\phi-1}\left(\frac{dy}{ds}\right)^2 - 2\phi t^{-16\phi^2+4\phi-1}\left(\frac{dz}{dx}\right)^2 - 8\phi^2 t^{-1}\left(\frac{dt}{dx}\right)^2 = 0$$

If a particle is initially at rest, that is, if

(3.6) $$\frac{dx}{ds} = \frac{dy}{ds} = \frac{dz}{ds} = 0$$

Then from equations of geodesic we find that for all such particles the components of spatial acceleration would vanish, namely.

(3.7) $$\frac{d^2x}{ds^2} = \frac{d^2y}{ds^2} = \frac{d^2z}{ds^2} = 0$$

and the particle would remain permanently at rest.

4. Conclusion

Here we have studied a non-static cosmological model in presence of electromagnetic field. The requirement that the conductivity be +ve puts an additional condition on metric potentials. Further we find that the four current, the electric field, density, pressure and expansion all start with in finite values at the initial singularity (t = 0) and tends toward zero when $t \to \infty$. Our investigation made in this paper is applicable and relevant in the study of hydrodynamic processes in heavenly bodies

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